Maths Club Feb 2013

Question Sheet 1

1. Multiply 13_{10} and 10_{10} in a base of your choice - Convert your answer back to check you get the right answer!

2. Make a multiplication table for a base of your choice.

   e.g. for base 4:

	1	2	3	10
1	1	2	3	10
2	2	10	12	20
3	3	12	21	30
10	10	20	30	100

3. What do you notice when multiplying by $(n-1)$, where n is your base (hint: what do you know about multiples of 9?)

4. What does 0.1_5 mean? how about 4.3_5? 2.76_8? (hint: what operation is being performed on place value as you move right?)

5. What does a base 1 representation look like? have you seen this anywhere before?

6. How would a negative base work? try representing numbers 1-15 in base -2. Is it possible to represent all numbers using negative bases?

7. [BONUS] How would you represent numbers in base $\sqrt{2}$? (hint: start with a representation of a number in base 2)

8. [BONUS] Golden ratio φ is defined by $\varphi+1 = \varphi^2$. can you represent all numbers 1-10 using base φ? (hint: $\varphi = \dfrac{1+\sqrt{5}}{2}$)

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Question Sheet 2

1. Use Peasant's multiplication to multiply 13 and 7. How many shifts are required, and how many additions? How does this compare to the Classical algorithm in base 10 or base 2?

2. Multiply 21 and 76 using the Classical algorithm, then 334 and 146. how many mutiplication, addition, shift operations have to happen? What is the most number of operations that may be required when multiplying numbers two of size 2? 3?

3. Multiply 15_{10} and 12_{10} using the Classical algorithm, and then in base 2 using the classical algorithm. how many operations need to be performed? would a larger base be better or worse?

4. Multiply 2436*5376 using the karatsuba algorithm, and 122345*231458. how many operations are required? What is the most number of operations that may be required when multiplying numbers of size 2? 4?

5. Multiply 101101_2 and 110111_2 using the classical algorithm, and by the karatsuba algorithm. how many multiplication, addition and shift operations are required?

6. [BONUS] multiply 101101_2 and 110111_2 using a method similar to the karatsuba algorithm, but instead partitioning the number into three sections (i.e. 10,11,01 and 11,01,11)

7. [BONUS] How long would it take to multiply two numbers of length N using the classical algorithm?

8. [BONUS] How long would it take to multiply two numbers of length N using the karatsuba algorithm?